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## ABSTRACT

### The Farm, the City and the Emergence of Social Security\*

During the period from 1880 to 1950 publicly managed retirement security programs became an important part of the social fabric in most advanced economies. In this paper we study the social, demographic and economic origins of social security. We describe a model economy in which demographics, technology, and social security are linked together. We study an economy with two locations (sectors), the farm (agricultural) and the city (industrial). The decision to migrate from rural to urban locations is endogenous and linked to productivity differences between the two locations and survival probabilities. Furthermore, the level of social security is determined by majority voting. We show that a calibrated version of this economy is consistent with the historical transformation in the United States. Initially a majority of voters live on the farm and do not want to implement social security. Once a majority of the voters move to the city, the median voter prefers a positive social security tax, and social security emerges.

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# 1 Introduction

Social security systems - structures that build some version of old-age pension support into the social fabric - are comparatively recent inventions. They emerged between 1880 and 1950 spreading from Scandinavia and Germany to Canada and the United States.<sup>1</sup> The notion that economic security in old-age had to be addressed by public policy is an assumption that transformed both the economic and social fabric of the countries involved. It is clearly not accidental that the need for publicly managed, rather than individually managed, economic security in old age arose in this set of countries during this period. As we show later in this paper, this need arose from the convergence of three important elements: an increase in the life expectancy of older generations, the shift from rural to urban social structures, and the productivity increases associated with industrialization.

In this paper we study the social, economic, and demographic origins of social security. We look to the past, to the impetus for the creation of the social security consensus as a means of understanding the shared challenges that these very countries, initiators of social security systems, are now facing as demographic trends place ever increasing pressure on pay-as-you-go (PAYG) pension systems.

In the United States social security was introduced in 1935 and the size of the program was expanded significantly in the 1940's and 1960's. There is a significant literature on the political economy of social security systems (Cooley and Soares 1996 and 1998, Conesa and Krueger 1999, Galasso 1999, Boldrin and Rustichini 2000, and Gonzales-Eiras and Niepelt 2005 e.g.) that analyses the political sustainability of PAYG social security systems.<sup>2</sup> The conclusion of most of this literature is that support for social security in democratic societies depends on the age of the median voter. As a population ages, the median voter is older and more likely to sustain social security. This literature, unfortunately, has nothing to say

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<sup>1</sup> See Lindert (1994) for the historical rise in public spending in pensions.

<sup>2</sup> There also exists a large literature that analyzes macroeconomic and distributional implications of the current social security system without political economy considerations (e.g. Imrohoroglu, Imrohoroglu and Joines 1985).

about the set of demographic, social, and economic conditions that led to the introduction of publicly managed old-age security.

One important background factor in the rise of social security is the change in the structure of economies over this period (see Scheiber and Shoven 1999). The implementation of social security was coincident with the change from primarily agricultural and rural economies to primarily urban and industrial economies. Figure 1 shows the change in rural/urban population mix for the U.S. from 1800 to 1940.<sup>3</sup> In the beginning of the 19th century almost everyone was living in rural areas, and nonfarm employment was almost non-existent. The share of population living in rural areas declined to 43.5% by 1940, while the farm populations was only 23% of the total US population. We argue that this major structural shift is important for thinking about the rise of social security because the rural/urban shift has implications for the provision of income for those who survive to old age.<sup>4</sup> As people migrated from the farm to the city, they could no longer rely on land as a source of old-age security, and political support for social security increased.

What were the economic and demographic forces that led to this shift from rural to urban population? Among the candidate answers to this question is the increase in the city wage relative to the farm wage that arose from greater technical change in the city relative to the farm. GDP per person employed increased by a factor of 3.5 in the U.S. between 1870 and 1940 (Madison 2001). While productivity in both the agriculture and the non-agricultural sectors grew rapidly during this period, the growth in non-agricultural sectors was faster than the growth in agriculture, leading to the transformation of the U.S. economy (see Hansen and Prescott 2002 and Greenwood and Uysal 2005). Figure 2 shows the change in total factor productivity (TFP) in agricultural and non-agricultural sectors in the U.S.<sup>5</sup>

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<sup>3</sup> Source: Hernandez (1996), Figure 5.

<sup>4</sup> Although the Great Depression is often considered as a major force behind the social security legislation in the U.S., its effects are far from clear. Miron and Weil (1998) conclude their study on the origins of social security by stating that: “Regarding the lasting impact of the Great Depression, our conclusion is that there were surprisingly little.” (page 321).

Between 1800 and 1940, TFP grew by a factor of 1.92 in agriculture, while it grew by a factor of 4.21 in manufacturing.

Another impetus for rural-urban migration was the increase in life expectancy. As life expectancy increased, two important changes occurred in the agricultural sector. First, the amount of farm labor relative to farm land rose, causing farm wages to fall. Second, as farmers lived longer, the transfer of land ownership via inheritance was delayed. Both events increased the relative attractiveness of living in the city for farmers, and encouraged rural-urban migration. Of crucial importance for this story is not that life expectancy at birth increased, but that life expectancy conditional on reaching or getting near retirement age increased. Figure 3 shows the changes in conditional survival probabilities from age 60 to 65, from 65 to 70, from 70 to 75, and from 75 to 80. Survival probabilities increased by about 5 percentage points between 1850 and 1900, and by another 2 percentage points between 1900 and 1940.<sup>6</sup>

One cannot understand the emergence of social security without understanding how social changes, demographics, and technology are linked together. In this paper we describe a model economy that illustrates how social security can arise in this context. As in Hansen and Prescott (2002), we study an overlapping generations economy with two locations (sectors), agricultural and industrial.<sup>7</sup> Farm production requires capital, labor and land. Land is a fixed factor, so there are decreasing returns to labor. City production on the other hand requires capital and labor and exhibits constant returns to scale.<sup>8</sup> Agents in this economy live up to three periods, as young, middle aged and old. They face an exogenous probability

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<sup>5</sup> Source: Greenwood and Uysal (2005), Figure 9.

<sup>6</sup> Source: U.S. Department of Health, Education and Welfare (1964), and Haines (1998).

<sup>7</sup> There is a large literature on structural transformation and the declining role of agriculture in the development process, see among others Greenwood and Seshadri (2002) and Gollin, Parente and Rogerson (2002).

<sup>8</sup> Hansen and Prescott (2002) model the industrial revolution as a switch from a (Malthus) production technology with a fixed factor of production, land, to a (Solow) production technology, with no fixed factors. Parente and Prescott (2005) use a similar framework to study the evolution of international income levels since 1750.

of dying at the end of second period of their lives. Land is passed from one generation to another by inheritance. Each period young agents make a once and for all decision about where to live.<sup>9</sup> There is also a social security system that taxes the young and the middle aged and pays transfers to the old. The level of social security taxes is determined by majority voting.<sup>10</sup> In the initial steady state of this economy the relative productivity of the farm sector is high and survival probabilities are low. As a result, farm incomes are high relative to city incomes. All agents live on the farm, and land is an important source of income for the old. The median voter is a middle-aged farmer who prefers a zero social security tax. When the city becomes more productive, people start migrating, and the importance of land diminishes. Eventually, the median voter becomes a middle-aged city worker who prefers a positive social security tax.

In the next section we describe the economic environment. In Section 3 we discuss the economic equilibrium, given an exogenous political process. In Section 4 we describe how taxes are determined. We provide some analytical results for a simplified version of the model in Section 5. In Section 6 we describe the results of our simulations. We conclude in Section 7.

## 2 Environment

Consider the following one-good, two-sector overlapping generations model. In the first sector (or location), which we will call the *farm*, labor, capital and land are combined to produce output. In the second sector (or location), which we call the *city*, the same good is produced using only labor and capital.

Agents live a maximum of 3 periods, which we refer to as young, middle-aged and old,

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<sup>9</sup> Among recent models with an explicit location decision see Vandenbroucke (2003), Hassler et al (2005), and Klein and Ventura (2006).

<sup>10</sup> The current paper follows the recent literature on dynamic models of political economy; see among others Krusell, Quadrini, and Rios-Rull (1997), Krusell and Rios-Rull (1999), and Hassler et al (2003). Graziella (2006) studies long run changes in bequest tax within a two-sector (agriculture and manufacturing) dynamic political economy model.



and face a probability,  $\pi$ , of surviving from the second to the last period. The objective of a young person is to maximize

$$U(c_y, c_m, c_o) = u(c_y) + \beta u(c_m) + \beta^2 \pi u(c_o), \quad (1)$$

where  $c_i$ ,  $i \in \{y, m, o\}$ , denotes age- $i$  consumption, and  $u$  is continuous, strictly increasing and strictly concave.

Each period every middle-aged person has a child who is born into the same location. When an agent is born on the farm, he makes a once-and-for-all decision to stay there or move to the city. Those who are born in the city are not allowed to move to the farm. The middle-aged and old agents can't change their locations.<sup>11</sup> Let the fraction of young, middle-aged and old agents who live on the farm be denoted by  $\lambda_y$ ,  $\lambda_m$  and  $\lambda_o$ , respectively.

In both locations young, middle-aged and old all inelastically supply one unit of labor.<sup>12</sup>

Each agent is born without any assets (capital or land) and is endowed with location dependent efficiency units  $\varepsilon_i^j$ ,  $j \in \{f, c\}$  and  $i \in \{y, m, o\}$ . Since only a fraction  $\pi$  of middle-aged people survive to old age, the total labor supply on the farm is given by  $N^f = \lambda_y \varepsilon_y^f + \lambda_m \varepsilon_m^f + \lambda_o \varepsilon_o^f \pi$  and the total labor supply in the city by  $N^c = (1 - \lambda_y) \varepsilon_y^c + (1 - \lambda_m) \varepsilon_m^c + (1 - \lambda_o) \varepsilon_o^c \pi$ .

Each period, agents are located either in the city or on the farm and can only work in that sector. There is a competitive labor market in each location. Let  $w^j$  denote the wages in sector  $j$ . Then, the labor income of an age- $i$  agent in location  $j$  is  $w^j \varepsilon_i^j$  for  $i \in \{y, m, o\}$  and  $j \in \{f, c\}$ .

People are not allowed to borrow, but can accumulate capital and rent it to firms in either sector at a competitive rate,  $\rho$ . Capital moves costlessly between the farm and the city, so let  $r = \rho - \delta$  be the common rate of return to capital, where  $\delta \in [0, 1]$  is the common rate of capital depreciation. There is no market in which agents can buy and sell land. This

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<sup>11</sup> The vast majority of migration from the farm to the city consisted of young workers. (Schieber and Shoven (1999), p. 18, and U.S. Bureau of the Census (1975), pp. 139, 465)

<sup>12</sup> We therefore abstract from the rise in retirement (i.e. decline in the labor force participation of old) since 1850s. See Kopecky (2005) for a model with endogenous retirement that links this rise to the technological progress in the production of leisure goods.

assumption is discussed in detail below. On the farm, when an agent dies (at the end of the second or third period), his land is inherited by the oldest surviving descendant. Therefore, a fraction of the land is owned by the  $\pi\lambda_o$  surviving old, and the remainder is owned by the  $(1 - \pi)\lambda_m$  middle-aged who inherited land early. We normalize the total amount of land to 1, so each landholding farmer has  $\frac{1}{\pi\lambda_o + (1-\pi)\lambda_m}$  units of land. Farmers rent their land to firms at a competitive rate  $q$ .

In a similar fashion, in both locations some middle-aged agents receive accidental capital bequests from their parents. As a result, middle-aged agents differ in their asset and land holdings on the farm, while they only differ by their asset levels in the city. If a young farmer chooses to move to the city, he gives up all claims on his parent's land, and that land, upon his parent's death, is divided equally among the remaining land owners. However, he still receives any accidental bequest his parent might leave, as we assume capital can freely move between the farm and the city.

Each sector is populated by a large number of production units which have access to constant returns to scale production functions represented by

$$Y^f = \gamma^f F^f(K^f, N^f, L), \quad (2)$$

and

$$Y^c = \gamma^c F^c(K^c, N^c), \quad (3)$$

where variables  $Y^j, K^j, N^j$  and  $L$ ,  $j \in \{f, c\}$ , refer to output, capital, and labor employed in each sector, and land used in the farm sector, respectively. The parameter  $\gamma^j$ ,  $j \in \{f, c\}$ , is the total factor productivity (TFP) in sector  $j$ . Land is a fixed factor and used only in the farm sector. We normalize the stock of land to one,  $L = 1$ .

Given the wage rate in sector  $j$ ,  $w^j$ , the rental rate for capital,  $\rho$  and the rental rate for land,  $q$ , the problem of a representative firm in the farm sector is given by

$$\max_{N^f, K^f, L} \{Y^f - w^f N^f - \rho K^f - qL\},$$

subject to (2), and in the city sector by

$$\max_{N^c, K^c} \{Y^c - w^c N^c - \rho K^c\},$$

subject to (3).

Finally, there is an economy-wide social security system that collects a lump-sum tax,  $\tau$ , from the young and the middle-aged and provides each old with an amount  $2\tau/\pi$ . The level of social security taxes are determined by majority voting in a way we detail below.

**Discussion** The assumption that land is passed from generation to generation is an important building block of our theory. First, there is evidence that inheritance was a major, perhaps the determining, factor of wealth inequality in the U.S. during the 19th century. According to Soltow (1982), in order to explain the relationship between age and wealth in the U.S. in 1870, inheritance was a much more significant factor than savings and capital accumulation over the life-cycle. Second, land was the most important component of inheritance in the 19th century. In his study of Butler County (Ohio), Newell (1986) documents that for 1803-1865 period, inheritance almost exclusively consisted of real property. Finally, according to Greven (1970) and Newell (1986), inheritance was the main way to acquire land in the settled areas.

Therefore, the picture that emerges is one in which people waited until their parents' death to obtain land and in which land provided an important source of wealth and income for the elderly. Available evidence, although limited, indicates that towards the end of 18th century, people 46 years or older held about 30% more wealth than the average in New England colonies (see Williamson and Lindert 1980, Table 1.2). More detailed data for the 19th century indicates a similar pattern. In 1850, those 60 years or older had about three times as much real estate wealth as the 30-39 age group (see Williamson and Lindert 1980, Table 1.7) and an analogous picture emerges for total wealth in 1870 (see Soltow 1992, Table 3.2). It is therefore not surprising that Scheiber and Shoven (1999) conclude that the over-65 age cohort controlled more wealth than any other group in the early 19th century.

### 3 Economic Equilibrium

At any point in time, the aggregate state in this economy consists of the distribution of capital across agents, the distribution of agents across the city and the farm, a social security tax, and an indicator variable of whether or not a social security system has operated in the past. The role of this indicator function will become clear once we define the political equilibrium below. In this section, we assume that agents take the political state as given.

Since agents are born without any capital, capital is owned by the middle-aged and the old. Furthermore, because they make different decisions, it is convenient to differentiate between the asset distribution of landed- and landless-middle-aged farmers. We represent the distribution of capital across old city and farm residents by  $\psi_o^c$  and  $\psi_o^f$ , and middle-aged city residents and farmers by  $\psi_m^c$  and  $\psi_m^{f\kappa}$  with  $\kappa = 1, 0$  indicating whether a middle-aged farmer is landed,  $\kappa = 1$ , or landless,  $\kappa = 0$ . In what follows we use  $\Psi = (\psi_m^c, \psi_o^c, \psi_m^{f1}, \psi_m^{f0}, \psi_o^f)$  to represent the set of distributions. We represent the distribution of agents between the two locations, city and farm, by  $\Lambda = (\lambda_y, \lambda_m, \lambda_o)$  where  $\lambda_j$  is the fraction of age- $j$  agents who live on the farm. Finally, we use  $S = (\Psi, \Lambda, \tau, h)$  to represent the aggregate state, where  $h$  is an indicator of whether or not a social security system ever operated in the past. If  $h = 1$  there was a system sometime in the past, if  $h = 0$ , there wasn't.

We represent the evolution of the aggregate state by three separate functions. First, we let  $\Psi' = G(S)$  represent next period's asset distribution given the current state  $S$ . Second we let  $\Lambda' = H(S)$  represent next period's distribution of agents across locations. In this section we describe the recursive competitive equilibrium given an exogenous policy rule that determines the law of motion for  $\tau$  and  $h$ . In particular, we assume that next period's social security tax level,  $\tau'$ , and history indicator  $h'$ , are given by  $(\tau', h') = P(\Psi, \Lambda, \tau, h)$ , and agents take the policy rule  $P$  as given when making their economic decisions. Once we define a recursive competitive equilibrium for a given  $P$  and show how  $G$  and  $H$  are determined, we describe how  $P$  is determined by the political process.

### 3.1 City Problem

We start by describing the economic problem of agents in the city. We approach agents' problems recursively, starting from the problem of an old agent, whose state consists of the aggregate state,  $S = (\Psi, \Lambda, \tau, h)$ , and his individual asset level  $a$ . An old agent in the city has three sources of income: labor income  $w^c \varepsilon_o^c$ , asset income  $(1 + r)a$ , and social security income  $\frac{2\tau}{\pi}$ . Let  $V_o^c(a, S)$  denote the value of being an old person with asset level of  $a$ . Since the old will simply consume their total resources, this is given by

$$V_o^c(a, S) = u(w^c \varepsilon_o^c + (1 + r)a + \frac{2\tau}{\pi}), \quad (4)$$

where for expositional clarity we suppress the dependence of  $w^c$  and  $r$  on aggregate state  $S$ .

Next, we look at the decision of middle-aged agents. Unlike the old, the middle-aged agents do not receive any social security payments, and they have to pay social security taxes. They also have to decide how much to save for their old age. Their decisions are determined by

$$\begin{aligned} V_m^c(a, S) &= \max_{a'} \{u(w^c \varepsilon_m^c + (1 + r)a - \tau - a') + \beta \pi V_o^c(a', S')\}, \\ s.t. \ S' &= (G(S), H(S), P(S)), \end{aligned} \quad (5)$$

where next period's asset distribution,  $\Psi'$ , and next period's distribution of agents between the two locations,  $\Lambda'$ , are determined by transition functions  $G(S)$  and  $H(S)$ , while next period's policy,  $(\tau', h')$ , is determined via  $P(S)$ . Let  $a_m^c(a, S)$  denote the savings decision of a middle-aged-city person with individual asset level  $a$  that results from problem (5).

Finally, we consider the decisions of the young agents who are born in the city. They are born with no assets. They might, however, get an (accidental) bequest next period if their parent does not survive to old-age. Let  $b(a, S)$  denote the bequest a young agent *expects* to get if his middle-aged parent has assets,  $a$ , and dies before reaching old age. The problem of a young agent is then given by

$$\begin{aligned}
V_y^c(b(a, S), S) &= \max_{a'} \{u(w^c \varepsilon_y^c - \tau - a') + \beta \pi V_m^c(a', S') \\
&\quad + \beta(1 - \pi) V_m^c(a' + b(a, S), S')\}, \\
s.t. \ S' &= (G(S), H(S), P(S)).
\end{aligned} \tag{6}$$

Note that a young agent will get  $b$  only if his parent dies and that happens with probability  $1 - \pi$ . Note also that the asset level of middle-aged agents provides enough information to determine next period's assets since it determines both the middle-aged as well as the young agents' savings decision. Let  $a_y^c(b(a, S), S)$  be the savings decisions of a young agent who expects to get  $b(a, S)$  as a bequest next period.

### 3.2 Farm Problem

The problem of an old agent on the farm is similar to the old agent's problem in the city, except the old farmer earns land income. His problem is given by

$$V_o^f(a, S) = u \left( w^f \varepsilon_o^f + (1 + r)a + \frac{q}{\pi \lambda_o + (1 - \pi) \lambda_m} + \frac{2\tau}{\pi} \right), \tag{7}$$

where  $\frac{1}{\pi \lambda_o + (1 - \pi) \lambda_m}$  is the per capita amount of land on the farm, and as in equation (4), we suppress the dependence of prices, including  $q$ , on  $S$ .

The problem of middle-aged agents on the farm is more complicated than that of those in the city, since in contrast to the city the middle-aged agents on the farm differ in landholding status. They are either landed or landless. The middle-aged farmer's problem can be written, for  $\kappa = 0, 1$ , as

$$\begin{aligned}
V_m^{f\kappa}(a, S) &= \max_{a'} \left\{ u \left( w^f \varepsilon_m^f + (1 + r)a + \frac{q\kappa}{\pi \lambda_o + (1 - \pi) \lambda_m} - \tau - a' \right) \right. \\
&\quad \left. + \beta \pi V_o^f(a', S') \right\}, \\
s.t. \ S' &= (G(S), H(S), P(S)).
\end{aligned} \tag{8}$$

Let  $a_m^{f\kappa}(a, S)$  be the decision rule for middle-aged farmers.

When considering the young farmer's saving decision, we need to do so jointly with his location decision. Some young farmers may stay on the farm, and some young farmers may move to the city. Their savings decisions will depend on where they choose to live. First consider a young farmer who *stays* on the farm. He might get a capital bequest of  $b^\kappa(a, S)$  as well as a land bequest from his parent, and he solves

$$\begin{aligned} V_y^{f\kappa s}(b^\kappa(a, S), S) &= \max_{a'} \{u(w^f \varepsilon_y^f - \tau - a') + \beta \pi V_m^{f0}(a', S') + \\ &\quad \beta(1 - \pi) V_m^{f1}(a' + b^\kappa(a, S), S')\}, \\ \text{s.t. } S' &= (G(S), H(S), P(S)). \end{aligned} \quad (9)$$

Let his decision be represented by  $a' = a_y^{f\kappa s}(b^\kappa(a, S), S)$ . Next consider a young farmer who *goes* to the city. He can only get a capital bequest of  $b^\kappa(a, S)$  from his parent and solves

$$\begin{aligned} V_y^{f\kappa g}(b^\kappa(a, S), S) &= \max_{a'} \{u(w^c \varepsilon_y^c - \tau - a') + \beta \pi V_m^c(a', S') + \\ &\quad \beta(1 - \pi) V_m^c(a' + b^\kappa(a, S), S')\}, \\ \text{s.t. } S' &= (G(S), H(S), P(S)). \end{aligned} \quad (10)$$

Let his decision be given by  $a' = a_y^{f\kappa g}(b^\kappa(a, S), S)$ .

Finally, let  $L(b^\kappa(a, S), S)$  be an indicator of whether the farmer is a goer or a stayer, which is simply determined by comparing his expected lifetime utility in each location, i.e.

$$L(b^\kappa(a, S), S) = \begin{cases} 1, & \text{if } V_y^{f\kappa g}(b^\kappa(a, S), S) \geq V_y^{f\kappa s}(b^\kappa(a, S), S) \\ 0, & \text{otherwise} \end{cases}. \quad (11)$$

### 3.3 Updating and Aggregation

When individuals solve their problems, they take the transition functions  $G$ ,  $H$ , and  $P$  as given. While we treat  $P$  as an exogenous function in this section, the other two transition functions,  $G$  and  $H$ , must be consistent with individual decisions in equilibrium. In this

section we analyze how the savings and location decisions of agents determine the evolution of aggregate assets and the fraction of agents living in each location.

We begin with the evolution of aggregate assets in the economy. In this economy, assets are owned either by the old or by middle-aged agents. Hence, given  $\psi_m^c(a)$  and  $\psi_o^c(a)$ , the current level of aggregate assets in the city,  $A^c$ , is simply

$$A^c = (1 - \lambda_m) \int a d\psi_m^c(a) + (1 - \lambda_o) \int a d\psi_o^c(a). \quad (12)$$

Similarly, the aggregate asset level on the farm,  $A^f$ , is

$$A^f = (1 - \pi)\lambda_m \int a d\psi_m^{f1}(a) + \pi\lambda_m \int a d\psi_m^{f0}(a) + \lambda_o \int a d\psi_o^f(a). \quad (13)$$

Given the particular demographic structure we have imposed, in order to determine the aggregate assets next period, all we need to know is the asset distribution of the middle-aged agents. To see this, note that next period's aggregate assets are determined by the savings decisions of young and middle-aged agents. Since the savings decisions of the young depend on the bequests they expect and these bequests are determined by the savings of the middle-aged agents, in order to find next period's aggregate asset level  $A^c$ ,  $\psi_m^c(a)$  and  $\psi_m^{f\kappa}(a)$  provide sufficient information. In particular, next period's aggregate asset level in the city is given by

$$\begin{aligned} A^{c'} &= (1 - \lambda_m) \int [a_y^c(a_m^c(a, S), S) + a_m^c(a, S)] d\psi_m^c(a) \\ &\quad + \lambda_m \left[ \int L(a_m^{f0}(a, S), S) a_y^{f0g}(a_m^{f0}(a, S), S) d\psi_m^{f0}(a) \right. \\ &\quad \left. + \int L(a_m^{f1}(a, S), S) a_y^{f1g}(a_m^{f1}(a, S), S) d\psi_m^{f1}(a) \right]. \end{aligned} \quad (14)$$

The first line in this equation is the portion of next period's assets that is determined by the savings decisions of the agents in the city. Here  $\int a_m^c(a, S) d\psi_m^c(a)$  gives the total savings of the middle-aged agents. These savings are either carried to their old age, or left as accidental bequests and constitute part of the assets owned by middle-aged agents next period. The term  $\int a_y^c(a_m^c(a, S), S) d\psi_m^c(a)$  is the other part of the assets owned by



middle-aged agents next period. It captures the savings done by the young, who in equilibrium anticipate correctly that they will receive  $a_m^c(a, S)$  as bequests. The next two lines capture the part of aggregate assets in the city that come from young agents who just moved to the city. The savings decisions of these newcomers depend on their parent's asset and land holding status, and are different from those of the young agents who are born in the city. Hence, if a young farmer whose parent has  $a$  units of assets and no land decides to go to the city, then  $L(a_m^{f0}(a, S), S) = 1$  and he saves  $a_y^{f0g}(a_m^{f0}(a, S), S)$ . The term  $\int L(a_m^{f0}(a, S), S) a_y^{f0g}(a_m^{f0}(a, S), S) d\psi_m^{f0}(a)$  is the aggregation of such assets.

In a similar fashion, next period's aggregate asset level on the farm is also determined by the asset distribution of landed- and landless-middle-aged agents and by the location decisions of the young. It is given by

$$A^{f'} = \lambda_m \left[ \int [(1 - L(a_m^{f0}(a, S), S)) a_y^{f0s}(a_m^{f0}(a, S), S) + a_m^{f0}(a, S)] d\psi_m^{f0}(a) + \right. \quad (15) \\ \left. \int [(1 - L(a_m^{f1}(a, S), S)) a_y^{f1s}(a_m^{f1}(a, S), S) + a_m^{f1}(a, S)] d\psi_m^{f1}(a) \right]$$

Like equation (14), the terms  $\int a_m^{f0}(a, S) d\psi_m^{f0}(a)$  and  $\int a_m^{f1}(a, S) d\psi_m^{f1}(a)$  represent the total savings of the middle-aged landless and landed agents, respectively, while the terms  $\int a_y^{f0s}(a_m^{f0}(a, S), S) d\psi_m^{f0}(a)$  and  $\int a_y^{f1s}(a_m^{f1}(a, S), S) d\psi_m^{f1}(a)$  are the savings done by the young who choose to stay on the farm.

Next, we describe how  $G$  is determined. This entails updating  $\psi_m^c(a)$ ,  $\psi_o^c(a)$ ,  $\psi_m^{f1}(a)$ ,  $\psi_m^{f0}(a)$  and  $\psi_o^f(a)$  in a manner that is consistent with the savings behavior of individuals. To this end, let  $Q = [0, \bar{a}]$  be the set of possible asset holdings for an individual in this economy. First, consider next period's asset distribution among the old in the city. This distribution will be determined by the savings of the current middle-aged agents in the city who survive to the next period. Then, it must be the case that for all  $\tilde{a} \in Q$ ,

$$\psi_o^{c'}(\tilde{a}) = \pi \int_Q I\{a_m^c(a, S) = \tilde{a}\} d\psi_m^c(a), \quad (16)$$

where  $I(\cdot) = 1$  if  $a_m^c(a, S) = \tilde{a}$ , and 0, otherwise. Similarly, the asset distribution of the old

on the farm is

$$\psi_o^{f'}(\tilde{a}) = \pi \int_Q I\{a_m^{f0}(a, S) = \tilde{a}\} d\psi_m^{f0}(a) + \pi \int_Q I\{a_m^{f1}(a, S) = \tilde{a}\} d\psi_m^{f1}(a), \quad (17)$$

where, with some abuse of notation, we use  $I$  as the appropriate indicator function.

Next period's asset distribution among the middle-aged agents in the city is determined by the location and savings decisions of young agents. One complication is that not all young agents make the same savings decisions. While some of them are born in the city, others move to the city this period. Furthermore, some of those movers had landless parents and some had landed parents. The following equation lists each of these cases:

$$\begin{aligned} \psi_m^{c'}(\tilde{a}) = & \int_Q [\pi I\{a_y^c(a_m^c(a, S), S) = \tilde{a}\} + (1 - \pi) I\{a_y^c(a_m^c(a, S), S) + a_m^c(a, S) = \tilde{a}\}] d\psi_m^c(a) \\ & + L(a_m^{f0}(a, S), S) \int_Q [\pi I_\pi^0\{a_y^{f0g}(a_m^{f0}(a, S), S) = \tilde{a}\} \\ & + (1 - \pi) I_{1-\pi}^0\{a_y^{f0g}(a_m^{f0}(a, S), S) + a_m^{f0}(a, S) = \tilde{a}\}] d\psi_m^{f0}(a) \\ & + L(a_m^{f1}(a, S), S) \int_Q [\pi I_\pi^1\{a_y^{f1g}(a_m^{f1}(a, S), S) = \tilde{a}\} \\ & + (1 - \pi) I_{1-\pi}^1\{a_y^{f1g}(a_m^{f1}(a, S), S) + a_m^{f1}(a, S) = \tilde{a}\}] d\psi_m^{f1}(a). \end{aligned} \quad (18)$$

The first line represents the total assets held by next period's middle-aged agents, who are young this period and were also born in the city. Their savings decisions are given by  $a_y^c(a_m^c(a, S), S)$ . If they do not receive any bequest, which happens with probability  $\pi$ , these are all the assets they have. There is however a  $1 - \pi$  chance that they receive a bequest. In this case, their total assets consist of their own savings and their parent's assets, and are given by  $a_y^c(a_m^c(a, S), S) + a_m^c(a, S)$ . The next two lines consider the same cases for young agents who go to the city and have landless parents, while the last two rows do the same for those who go to the city and have landed parents.

Finally, next period's asset distribution for middle-aged agents on the farm is given by the savings decisions of the young who choose to stay there. For the landless-middle-aged

farmers we have,

$$\begin{aligned} \psi_m^{f0'}(\tilde{a}) = & \pi[(1 - L(a_m^{f0}(a, S), S)) \int_Q I_\pi^0 \{a_y^{f0s}(a_m^{f0}(a, S), S) = \tilde{a}\} d\psi_m^{f0}(a) \\ & + (1 - L(a_m^{f1}(a, S), S)) \int_Q I_\pi^1 \{a_y^{f1s}(a_m^{f1}(a, S), S) = \tilde{a}\} d\psi_m^{f1}(a)]. \end{aligned} \quad (19)$$

And, for the landed-middle-aged farmers we have,

$$\begin{aligned} \psi_m^{f1'}(\tilde{a}) = & (1 - \pi)[(1 - L(a_m^{f0}(a, S), S)) \int_Q I_{1-\pi}^0 \{a_y^{f0s}(a_m^{f0}(a, S), S) + a_m^{f0}(a, S) = \tilde{a}\} d\psi_m^{f0}(a) \\ & + (1 - L(a_m^{f1}(a, S), S)) \int_Q I_{1-\pi}^1 \{a_y^{f1s}(a_m^{f1}(a, S), S) + a_m^{f1}(a, S) = \tilde{a}\} d\psi_m^{f1}(a)]. \end{aligned} \quad (20)$$

Next, in order to determine  $H$ , we consider how the location decisions are updated. Suppose the current location decisions of agents are given by  $\Lambda = (\lambda_y, \lambda_m, \lambda_o)$ . Since all young agents survive to middle age, it must be the case that  $\lambda'_m = \lambda_y$ . Similarly, since the survival probability,  $\pi$ , is identical in both locations,  $\lambda'_o = \lambda_m$ . The fraction of young agents who will be on the farm, however, depends on the location decisions of those agents who are born on the farm. A fraction  $\lambda_y$  will be born on the farm next period. Yet, according to equation (11), some of them will move to the city. Hence, for any  $S'$ , the total fraction who stay, among those whose parent does not have any land, is given by  $\int (1 - L(a_m^{f0}(a, S'), S')) d\psi_m^{f0'}(a)$ . The same expression for those whose parent has land is given by  $\int (1 - L(a_m^{f1}(a, S'), S')) d\psi_m^{f1'}(a)$ . Putting these pieces together implies the following consistency condition for  $\Lambda'$

$$\begin{aligned} \Lambda' = & \left( \lambda_y \left[ \pi \int (1 - L(a_m^{f0}(a, S'), S')) d\psi_m^{f0'}(a) \right. \right. \\ & \left. \left. + (1 - \pi) \int (1 - L(a_m^{f1}(a, S'), S')) d\psi_m^{f1'}(a) \right], \lambda_y, \lambda_m \right). \end{aligned} \quad (21)$$

### 3.4 Economic Equilibrium

Given a policy function  $P(S)$ , a *recursive competitive equilibrium* for this economy consists of a set of value functions,  $V_y^c(b(a, S), S)$ ,  $V_m^c(a, S)$ , and  $V_o^c(a, S)$ , for agents who live in the

city and  $V_y^{f\kappa s}(a, S)$ ,  $V_y^{f\kappa g}(a, S)$ ,  $V_m^{f\kappa}(b^\kappa(a, S), S)$   $\kappa = 0, 1$ , and  $V_o^f(a, S)$  for agents who live on the farm; a set of decision rules  $a_y^c(b^c(a, S), S)$  and  $a_m^c(a, S)$  for agents who live in the city, and  $a_y^{f\kappa s}(b^\kappa(a, S), S)$ ,  $a_y^{f\kappa g}(b^\kappa(a, S), S)$  and  $a_m^{f\kappa}(a, S)$ ,  $\kappa = 0, 1$ , for agents who live on the farm; a location rule for young farmers,  $L(b^\kappa(a, S), S)$ ,  $\kappa = 0, 1$ ; a set of pricing functions  $r(S)$ ,  $w^c(S)$ ,  $w^f(S)$ , and  $q(S)$ , and a set of aggregate laws of motion  $H(S)$  and  $G(S)$  such that:

- Given the transition functions  $P(S)$ ,  $H(S)$ , and  $G(S)$ , and pricing functions  $r(S)$ ,  $w^c(S)$ ,  $w^f(S)$ , and  $q(S)$ , the value functions and corresponding decision rules solve the appropriate household problems in equations (4), (5), (6), (7), (8), (9), (10), and (11), with  $b(a, S) = a_m^c(a, S)$  and  $b^\kappa(a, S) = a_m^{f\kappa}(a, S)$ ,  $\kappa = 0, 1$ .
- The pricing functions,  $r(S)$ ,  $w^c(S)$ ,  $w^f(S)$ , and  $q(S)$ , are determined by profit maximization of the representative firm in each sector together with a no arbitrage condition for capital, i.e.  $r(S)$ ,  $w^c(S)$ ,  $w^f(S)$ , and  $q(S)$  satisfy

$$\begin{aligned} w^c(S) &= F_2^c(K^c, N^c), \\ w^f(S) &= F_2^f(K^f, N^f, L), \\ q(S) &= F_3^f(K^f, N^f, L), \end{aligned}$$

and

$$r(S) + \delta = F_1^c(K^c, N^c) = F_1^f(K^f, N^f, L),$$

with aggregate labor and capital in each sector given by

$$\begin{aligned} N^f &= \lambda_y \varepsilon_y^f + \lambda_m \varepsilon_m^f + \lambda_o \pi \varepsilon_o^f, \\ N^c &= (1 - \lambda_y) \varepsilon_y^c + (1 - \lambda_m) \varepsilon_m^c + (1 - \lambda_o) \pi \varepsilon_o^c, \end{aligned}$$

and

$$K = K^c + K^f = A^c + A^f,$$

where  $A^c$  and  $A^f$  are given by equations (12) and (13), and  $K^c$  and  $K^f$  are determined by the no arbitrage condition.

- Aggregate transition functions are consistent with individual decisions: (i) The transition function  $G$  is consistent with individual savings decisions and is determined by equations (16), (17), (18), (19), and (20). (ii) The transition function  $H$  is consistent with individual location decisions and is determined by (21).

## 4 Political Equilibrium

So far we have taken the function  $P$  as given. We now focus on the social security taxes that are determined by equilibrium voting of successive generations. We assume sincere voting, i.e. that each agent votes for his most preferred alternative in each period. It is not obvious whether an equilibrium with social security can be supported as a political outcome in a democratic voting process. The current young and middle-aged do not benefit from the system, yet their support is critical. Indeed, the current young and middle-aged will always choose to pay nothing in the current period, as long as they believe that the system will be there for them in the future. To induce these agents to vote for social security we introduce the following *reputational* mechanism: if a majority of voters deviate from the current level of social security, then the system collapses.<sup>13</sup> This way, young and middle-aged workers balance the benefit of not paying into the system against the cost of not receiving anything from it in the future.

**Definition 1** *For any  $\tau > 0$ , we will say that a policy function  $P(S)$  is sustainable in state  $S = (\Psi, \Lambda, \tau, h)$ , if*

$$V^M(\Psi, \Lambda, \tau, h; P) \geq \mathcal{V}^M(\Psi, \Lambda),$$

*where  $V^M$  is the remaining lifetime utility of the median voter in an economy with current aggregate state  $S = (\Psi, \Lambda, \tau, h)$  and policy function  $P$ , and  $\mathcal{V}^M$  is the remaining lifetime utility of the median voter if social security is eliminated forever.*

The value  $\mathcal{V}^M(\Psi, \Lambda)$  only depends on  $\Psi$  and  $\Lambda$ , i.e. the aggregate state (aggregate physical capital and distribution of agents between the city and the farm) in which the social security tax is eliminated. In other words,  $P$  is sustainable in  $S$  if a majority of voters vote “yes” for

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<sup>13</sup> Two early papers that emphasized the political sustainability of social security were Browning (1975) and Sjoblom (1985).

keeping it today, instead of moving to an economy with no social security. Let the indicator function  $M(\Psi, \Lambda, \tau, h; P)$  denote the yes/no decision of the median voter, i.e.

$$M(\Psi, \Lambda, \tau, h; P) = \begin{cases} 1, & \text{if } V^M(\Psi, \Lambda, \tau, h; P) \geq \mathcal{V}^M(\Psi, \Lambda, 0, 1) \\ 0, & \text{otherwise} \end{cases}.$$

Obviously there can be many policy rules that are sustainable. In this paper we consider a variant of *constant* social security taxes: (i) if a social security system has never been in place, it may start at any point, (ii) once a system is operating, if it continues, the tax remains constant, (iii) if a system is dismantled, it cannot start up again, and the tax remains zero forever.

To define the policy function  $P$ , we begin by noting that the history,  $h$ , of the social security system is important for its future evolution. Specifically, if the current tax level is zero, it is either because no median voter has voted for social security, and a system is still a possibility, or a system existed in the past, was dismantled, and no possibility exists of a positive tax in the future.

If  $\tau = 0$ , and  $h = 1$ , then a social security system collapsed in the past, and cannot be restarted. Therefore,

$$P(\Psi, \Lambda, 0, 1) = (0, 1) \tag{22}$$

If  $\tau = 0$ , and  $h = 0$ , then a social security system has never been operative. It may, or may not begin next period, depending on the preferences of tomorrow's median voter. Therefore,

$$P(\Psi, \Lambda, 0, 0) = (\arg \max_{\tau} V^M(\Psi', \Lambda', \tau, 0; P), 0). \tag{23}$$

If the tax is strictly positive today, then either today or sometime in the past a median voter instituted his most preferred tax, call it  $\tau^* > 0$ . In this case, next period the system either continues at the same tax level or is dismantled. Regardless of the value of  $h$ ,  $h' = 1$ , because next period there will have been a social security system in the past. Hence,

$$P(\Psi, \Lambda, \tau^*, h) = \begin{cases} (\tau^*, 1), & \text{if } M(\Psi', \Lambda', \tau^*, 1; P) = 1 \\ (0, 1), & \text{if } M(\Psi', \Lambda', \tau^*, 1; P) = 0 \end{cases}. \tag{24}$$

A *political equilibrium* is then a recursive competitive equilibrium with the policy function  $P$  defined by equations (22), (23), and (24). Once a median voter sets  $\tau^* > 0$ , then future generations of median voters simply decide whether to sustain the system or not, knowing that once the system is dismantled, it is gone forever.<sup>14</sup> Obviously, the median voter who chooses  $\tau^* > 0$ , takes into account the reputational mechanism that is in effect. Finally, if  $h = 0$ , and the median voter prefers a zero tax, he considers the possibility that sometime in the future a social security system might be implemented.

At a general level, not much can be said analytically about this model. In the following sections we choose functional forms, assign parameter values, and perform numerical evaluations. Some valuable analytical insight can be gained, however, by focusing on a steady state economy without capital. This is what we turn to next.

## 5 Steady State Economy without Capital

Consider a steady state version of the economy outlined above, i.e. let  $\Lambda' = \Lambda, \Psi' = \Psi, h' = h$ , and  $\tau' = \tau$ . In the steady state there will be a constant fraction  $\lambda$  of population that lives on the farm, i.e.  $\lambda_y = \lambda_m = \lambda_o = \lambda$ . Suppose the farm sector uses only labor and land, while labor is the only factor of production in the city sector. In particular, let the production function in the farm sector be

$$Y^f = \gamma^f (N^f)^\mu (L)^{1-\mu}.$$

Hence, with  $L = 1$ , the rental rates are given by

$$w^f = \mu \gamma^f (N^f)^{\mu-1}, \tag{25}$$

and

$$q = (1 - \mu) \gamma^f (N^f)^\mu. \tag{26}$$

Since we are in a steady state, the aggregate labor on the farm is  $N^f = (\varepsilon_y^f + \varepsilon_m^f + \pi \varepsilon_o^f) \lambda$ . Note that since land is a fixed factor of production, there are decreasing returns to labor. As a

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<sup>14</sup> Once a median voter sets  $\tau^* > 0$ , the way we have defined a political equilibrium is similar to Cooley and Soares (1999).

result, when people live longer and the farm sector gets more crowded, i.e. when  $\pi$  increases,  $w^f$  declines and  $q$  increases. It is also the case that as people move out of agriculture the pressure on farm wages is reduced, since as  $\lambda$  declines,  $w^f$  rises and  $q$  declines. Let the production function in the city be

$$Y^c = \gamma^c N^c, \quad (27)$$

which implies  $w^c = \gamma^c$ . Finally, let

$$u(c) = \log(c). \quad (28)$$

Furthermore, suppose agents have access to a storage technology that transfers resources from current to future periods. In particular, suppose a unit of goods not consumed today becomes  $1 + r$  units of goods tomorrow.

We focus here on how taxes are determined and characterize the behavior of middle-aged agents, who are most likely to be median voters in equilibrium. It turns out that the amount a middle-aged person chooses to store depends critically on the social security tax. For each middle-aged agent there exists a threshold tax level. This threshold depends on his wealth, and if the existing social security tax is greater than this threshold, then he stores nothing, while if it is strictly less than this threshold, he stores a positive amount. Intuitively, a person's threshold tax level reflects how much of his resources he would like to transfer from today to tomorrow. If the actual tax level is lower than what he prefers, then he stores. Hence, the closer the actual tax level is to his threshold (or ideal) tax level, the less he needs to store in order to make up the difference. Furthermore, if a person saves, his savings decision is decreasing in the social security tax, increasing in his wealth, and increasing in the survival probability.

These results are formalized in the next proposition. All proofs are in the Appendix. To streamline the presentation it is helpful to define middle-age and old-age income variables. Let  $I_m^j$ ,  $j \in \{c, f0, f1\}$ , be pre-tax total labor and land incomes of the middle-aged. So,  $I_m^c = \varepsilon_m^c w^c$ ,  $I_m^{f0} = \varepsilon_m^f w^f$ , and  $I_m^{f1} = \varepsilon_m^f w^f + q/\lambda$ . Let  $I_o^j$ ,  $j \in \{c, f0, f1\}$  be total labor and land incomes of the old-age. So,  $I_o^c = \varepsilon_o^c w^c$ ,  $I_o^{f\kappa} = \varepsilon_o^f w^f + q/\lambda$ ,  $\kappa \in \{0, 1\}$ . Note that all old



farmers have the same labor and land incomes, regardless of their middle-age land status. However, it is easier in terms of exposition to separate them.

**Proposition 1** *Let  $p = (r, w^c, w^f, q)$ . Given  $p$  and  $\tau$ , for any middle-aged person of type  $j \in \{c, f0, f1\}$ , there exists a threshold tax level  $\hat{\tau}_m^j(a; p, \tau) \geq 0$  such that: (i) If  $\hat{\tau}_m^j(a; p, \tau) \leq \tau$ , then  $a_m^j(a; p, \tau) = 0$ . (ii) If  $\hat{\tau}_m^j(a; p, \tau) > \tau$ , then  $a_m^j(a; p, \tau) = \frac{\beta\pi(1+r)(I_m^j+a-\tau)-(I_o^j+2\tau/\pi)}{(1+r)(1+\beta\pi)} > 0$ ; and  $\frac{\partial a_m^j(a; p, \tau)}{\partial \tau} < 0$ ,  $\frac{\partial a_m^j(a; p, \tau)}{\partial a} > 0$ , and  $\frac{\partial a_m^j(a; p, \tau)}{\partial \pi} > 0$ .*

The next proposition provides a characterization of the threshold tax level  $\hat{\tau}$ . This threshold is increasing in the middle-aged agent's wealth, since an agent with higher wealth has more incentive to transfer his resources to old age. On the other hand, if the non-social security income of the old is sufficiently high relative to the pre-tax income of the middle-aged, then the reservation tax is zero. Agents who have enough resources when they are old do not want to transfer resources to their old age.

**Proposition 2** *Given  $p$  and  $\tau$ , for any middle-aged person of type  $j \in \{c, f0, f1\}$ : (i)  $\hat{\tau}_m^j(a; p, \tau) = \max\{0, \frac{\beta\pi(1+r)(I_m^j+a)-I_o^j}{2/\pi+\beta\pi(1+r)}\}$ ; (ii) If  $\hat{\tau}_m^j(a; p, \tau) > 0$ , then  $\frac{\partial \hat{\tau}_m^j(a; p, \tau)}{\partial a} > 0$ .*

Next, we consider the decision of the middle-aged median voter. Suppose the return to social security,  $2/\pi$ , is less than the return to storage,  $1+r$ . Then, the median voter chooses a zero tax, and any agent who wants to save, saves entirely through storage. If,  $2/\pi > 1+r$ , then if the median voter wants to save, he chooses a positive tax, saves entirely via social security, and stores nothing. Middle-aged agents who have higher wealth than the median voter, store positive amounts, since they want to save more than the median voter. What is key in both cases is whether or not middle-aged agents want to save. If they do not, then neither social security, nor storage will be operative in equilibrium. These results are outlined in the following proposition.

**Proposition 3** *Given  $p$ , let  $a$  be the stored assets of the median voter, let  $a_m^j$  be the storage decision of the middle-aged median voter of type  $j \in \{c, f0, f1\}$ , and let  $\tau^*$  denote his preferred tax rate. (i) If  $\frac{2}{\pi} < (1+r)$ , then  $\tau^* = 0$ , and  $a_m^j(a; p, \tau^*) = \max\{0, \frac{\beta\pi(1+r)(I_m^j+a)-I_o^j}{(1+r)(1+\beta\pi)}\}$ . (ii) If  $\frac{2}{\pi} > (1+r)$ , then  $\tau^* = \max\{0, \frac{2\beta(I_m^j+a)-I_o^j}{2(\beta+1/\pi)}\}$ , and  $a_m^j(a; p, \tau^*) = 0$ . (iii) If  $\frac{2}{\pi} = (1+r)$ , then  $\tau^* \in [0, \hat{\tau}^j]$ , and  $a_m^j(a; p, \tau^*) \in [0, \max\{0, \frac{\beta\pi(1+r)(I_m^j+a)-I_o^j}{(1+r)(1+\beta\pi)}\}]$ .*

## 5.1 Discussion

Land plays an important role in this framework for two reasons. First, it is a fixed factor on the farm, so increasing survival probabilities reduces farm wages. This crowding of land encourages young farmers to migrate to the city. Second, land provides insurance for farmers. The promise of land upon survival to old age for middle-aged-landless farmers creates a steep age-income profile that discourages saving. For reasonable parameters, this implies that as long as most people are on the farm, and the middle-aged-landless farmer is the median voter, there will be no social security. It is important for this result that there is no market for land, it is the inherited nature of land that creates this wedge in age-income profiles.<sup>15</sup>

The political economy aspect of the environment is simple, yet critical. Middle-aged agents only pay into the system one period, while their benefits are based on two periods of payments. This encourages support for social security, even when age-income profiles are flat. Land is not available for city workers as old age security. They earn only labor income when middle-aged and old. Therefore, they have age-income profiles that are relatively flatter than that of farmers, and thus are more likely to support social security. An important feature of this framework is that as the fraction of people living on the farm falls, the identity of the median voter shifts from the farm to the city and support for social security can emerge. As was highlighted in the last section, in order for social security to arise at all, the returns to the middle-aged voter,  $2/\pi$ , must be greater than the returns to saving,  $1 + r$ .

## 6 Economy with Capital

We are now ready to carry out our quantitative exercise and evaluate if a calibrated version of our model is consistent with the historical experience of the U.S. economy. Consider the general setup from Section 2 and assume that young and middle-aged agents save in the forms of risk-free, productive capital. Although the basic intuition from the analytical results of the previous section remains valid, there are now general equilibrium effects at

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<sup>15</sup> This result also depends on the return to land relative to the return to farm labor. With a higher share to farm labor, land plays a smaller role, causing the age-income profile of the landless farmer to flatten.

play as well. As a result, changes in relative productivity levels and survival probabilities will not only determine farm wages and land returns via migration, but will also affect all prices via changes in individual capital accumulation decisions. In their decisions about the social security system, agents still compare the return to capital with the return to social security, but the return to capital is now an endogenous variable.

In this section we show that a calibrated version of this economy can generate an initial steady state in which a majority of the population lives on the farm and the median voter chooses not to introduce a social security system, and a transition to a new steady state along which the median voter chooses a positive and sustainable social security tax. We interpret the initial steady as the U.S. economy in 1800 and the final one as the U.S. economy in 1940. Computing the transition is non-trivial. Not only do the capital stock and location choices (and hence prices) have to be consistent with individual asset accumulation and migration decisions, but the sequence of tax levels that individuals expect must be those that the median voter in each generation chooses. In order to develop quantitative implications of this model economy, we first choose functional forms for utility and production functions and assign parameter values.

As in the previous section, let the utility function be  $u(c) = \log(c)$ . Since the production side of our model economy closely follows Hansen and Prescott (2002), we borrow both functional forms and parameter values from them. In particular, we assume that the production function on the farm sector is given by

$$Y^f = \gamma^f [N^f]^\mu [K^f]^\phi [L]^{1-\mu-\phi},$$

and in the city sector it is

$$Y^c = \gamma^c [N^c]^{1-\theta} [K^c]^\theta.$$

These choices imply that

$$w^c = (1 - \theta)\gamma^c (N^c)^{-\theta} (K^c)^\theta, \tag{29}$$

$$w^f = \mu\gamma^f (N^f)^{\mu-1} (K^f)^\phi, \tag{30}$$

$$q = (1 - \mu - \phi)\gamma^f(N^f)^\mu(K^f)^\phi, \quad (31)$$

and

$$r = r^c = \theta\gamma^c(N^c)^{1-\theta}(K^c)^{\theta-1} - \delta = \phi\gamma^f(N^f)^\mu(K^f)^{\phi-1} - \delta = r^f. \quad (32)$$

The parameter values we use are  $\mu = 0.6$ ,  $\phi = 0.1$ , and  $\theta = 0.4$ .<sup>16</sup> We set the length of a model period to 20 years. We also assume that capital depreciates completely, i.e.  $\delta = 1$ , which is not critical for any of the results, but simplifies the computational burden.

Next we select the values for relative TFP levels and survival probabilities. We borrow TFP numbers from Greenwood and Uysal (2005). For the 1800 economy we set  $\gamma_{1800}^f = \gamma_{1800}^c = 1$ . Since the relative TFP values are the key determinants of migration decisions in the model, we keep  $\gamma_{1940}^f = 1$  and set  $\gamma_{1940}^c = 2.19$ . These choices imply that the relative TFP growth is as reported by Greenwood and Uysal (2005) and reproduced in Figure 2. Historical estimates for age-specific-mortality rates and life tables do not go back further than 1850 (see Haines 1988). In 1850, a 60 year-old man had about a 47% chance of living to his 80th birthday. Since available evidence does not indicate any significant improvement in mortality between 1800 and 1850, we set  $\pi_{1800} = 0.47$ .<sup>17</sup> In 1940 the chances that a 60 year old man saw his 80th birthday increased to about 56%. Therefore, we select  $\pi_{1940} = 0.56$ .<sup>18</sup>

Finally, we assume that agents have flat age-earning profiles both on the farm and in the city, i.e.  $\varepsilon_i^j = 1$  for  $j \in \{f, c\}$  and  $i \in \{y, m, o\}$ . Age-earning profiles in the 19th century did indeed differ from the usual hump-shaped pattern. According to Kaelble and Thomas (1991), incomes of working class household heads increased slightly between ages

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<sup>16</sup> The value for capital share in the city (industrial) technology,  $\theta = 0.4$ , is the standard value for the postwar U.S. economy. The labor share is assumed to be the same for both sectors,  $\mu = 1 - \theta = 0.6$ . Finally,  $\phi = 0.1$  is picked to be consistent with historical evidence on agricultural incomes. See Hansen and Prescott (2002) for details.

<sup>17</sup> According to Haines (1988), the crude death rate in New York City was as high in 1850 as it was in 1804 (see Figure 1, page 150). In many New England towns there was not much improvement in life expectancy at age 20 either (see Table 1, page 151).

<sup>18</sup> The data for 1850 are from Haines (1988) and for 1950 are taken from the U.S. Department of Health, Education and Welfare (1964). They are the average of the conditional survival probabilities from age 60 to 65, from 65 to 70, from 70 to 75 and 75 to 80 in 1850 and 1950, respectively. The 1850 numbers are for white males only and are based on West Model.

20 and 40, but were pretty much flat after age 40. These flat profiles were a common feature of agricultural workers as well as low skilled non-agricultural workers.<sup>19</sup> We make the strong assumption that age-earning profiles were also flat in the city. We consider this as a conservative assumption for our results, since a hump-shaped profile for city workers would simply increase the incentives of middle aged workers to shift resources to their old age and increase the political support for social security even further.

Note that we fix all these parameter values prior to running our simulations. We are left with only one more parameter to pick,  $\beta$ . We set  $\beta = 0.818$  (a yearly value of 0.99). This value implies that the yearly return to capital in the 1940 steady state is about 5.8%.<sup>20</sup> Table I summarizes our parameter choices.

Table I — Parameter Values

| $\beta$ | $\mu$ | $\phi$ | $\theta$ | $\delta$ | $\gamma_{1800}^f$ | $\gamma_{1800}^c$ | $\gamma_{1940}^f$ | $\gamma_{1940}^c$ | $\pi_{1800}$ | $\pi_{1940}$ |
|---------|-------|--------|----------|----------|-------------------|-------------------|-------------------|-------------------|--------------|--------------|
| 0.818   | 0.6   | 0.1    | 0.4      | 1        | 1                 | 1                 | 1                 | 2.19              | 0.47         | 0.56         |

## 6.1 Results

Table II shows the results for the two steady states, 1800 and 1940. First consider the economy in 1800. In our 1800 economy everyone lives on the farm,  $\lambda = 1$ . This is consistent with the U.S. experience. At that time, about 94% of population lived in rural areas, and the fraction of population working on the farm was possibly even higher (see Figure 1). In the 1800 steady state, the median voter is a landless-middle-aged farmer, who does not want social security, so the equilibrium value of  $\tau$  is zero. Notice that this happens even

<sup>19</sup> Doepke and Zilibotti (2005) contrast relatively flat wage profiles of agricultural workers and land owners with steep wage profiles of entrepreneurs in the 19th century. They model the emergence of capitalism within a model of structural transformation in which entrepreneurs influence their children’s preferences in an attempt to make them more patient.

<sup>20</sup> Hansen and Prescott (2002) target a 4-4.5 percent rate of return on capital for post-war period. Cooley and Prescott (1995) report a higher value, 6.9 percent. The value for the return on capital in our new steady state is right between these two values. See Gomme and Rupert (2005) for a recent discussion.

though  $2/\pi$  (about 4.25) is larger than  $1 + r$ , so the return to social security is greater than the return to capital. However, the middle-aged-landless farmer prefers to save nothing due to his steep age-income profile. Next, consider the 1940 economy. Now about 16% of the population lives on the farm, a value close to the 23% observed in the U.S. at that time (see Figure 1). This is quite remarkable since nothing in our parameter choices targets directly the fraction of agents living on the farm.

Consistent with historical experience, the return on capital is much higher in the new steady state, despite a more than fourfold increase in aggregate capital stock. In 1940, about 16% of the population lives on the farm, but a much smaller (about 4.3%) fraction of aggregate capital stock is allocated to farm production. Also consistent with historical evidence, the rental value of land declines significantly. In 1940 it is about one third of its 1800 value.<sup>21</sup> Lastly, note that while the returns per unit of land,  $q$ , fall, the returns to land for landholders,  $q/\lambda$ , actually rise, .40 to .54, which keeps them on the farm despite rising city wages.

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<sup>21</sup> According to Hansen and Prescott (2002), the value of U.S. farmland relative to GDP declined from 88% in 1870 to 20% in 1950 (see Table 2, page 1209).

TABLE II - Initial and Final Steady States

|              | 1800                       | 1940                    |
|--------------|----------------------------|-------------------------|
| $\tau$       | 0                          | 0.080                   |
| $\lambda$    | 1                          | 0.160                   |
| $1 + r$      | 2.449                      | 3.072                   |
| $w^f$        | 0.311                      | 0.551                   |
| $w^c$        | -                          | 0.569                   |
| $q$          | 0.384                      | 0.114                   |
| $q/\lambda$  | 0.384                      | 0.710                   |
| $K$          | 0.052                      | 0.280                   |
| $K^f$        | 0.052                      | 0.012                   |
| $K^c$        | -                          | 0.267                   |
| $N^f$        | 2.470                      | 0.413                   |
| $N^c$        | -                          | 2.167                   |
| Median Voter | middle-age-landless farmer | middle-aged city worker |

Table III and Figure 4 illustrate the transitional dynamics resulting from this exercise. We assume that the economy is at its 1800 steady state initially (period 0) and suddenly and unexpectedly productivity and life expectancy increase to their 1940 values. In the period of the change (period 1), the capital stock is fixed at its initial steady state level. However, due to the higher productivity in the city and the higher survival probability, the city is a much more attractive location for young farmers and many choose to migrate,  $\lambda_y = 0.66$ . This population shift alters the labor supply on the farm and in the city. Indeed, since a large fraction of population migrates in the first period of the transition, both farm and city wages rise. Given the rise in productivity levels, the return to capital, which is fixed at its old steady state value, increases significantly from 2.449 to 5.718. As people start moving away from the farm, the return to land starts to fall as well.

Because the migration only affects the location of the young, in period 1 the median voter is still a middle-aged landless farmer, who prefers no social security.<sup>22</sup> So, in the initial period of the change, the tax remains unchanged at 0. However, agents are aware that the mass migration of young farmers to the city will shift the identity of the median voter in the next period, and alter support for social security.

In the second period of the transition, the initial young migrants now become middle-aged-city workers, who support a positive (sustainable) level of social security,  $\tau = .08$ .<sup>23</sup> No further migration occurs this period, or over the remainder of the transition. However, the new steady state farm population takes three periods to attain, as the initial young migrants age. As the population reallocates between the two locations and people start accumulating capital, the return to capital falls to 3.718, and then converges to 3.072 in the new steady state.

In the new steady state, even though total labor supply in the city rises due to the increase in life expectancy and in city population, because of the increase in technological progress and in the aggregate capital stock, the city wage rises. There is no technological advance on the farm. But the out migration of farmers causes farm labor supply to fall, and so farm wages rise. And, while there is a large increase in the aggregate capital stock, the technological advance in the city, coupled with the increase in labor supply in the city, leads to an increase in the return to capital.

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<sup>22</sup> We computationally verify that preferences are single peaked.

<sup>23</sup> Again, we computationally verify that preferences are single peaked in the period of the vote for a positive tax.



TABLE III - Transition

|             | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\tau$      | 0     | 0     | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  |
| $\lambda_y$ | 1     | 0.660 | 0.349 | 0.160 | 0.160 | 0.160 | 0.160 | 0.160 |
| $1 + r$     | 2.449 | 5.718 | 3.718 | 3.443 | 3.203 | 3.126 | 3.088 | 3.072 |
| $w^f$       | 0.311 | 0.325 | 0.416 | 0.544 | 0.548 | 0.549 | 0.550 | 0.551 |
| $w^c$       | -     | 0.376 | 0.501 | 0.528 | 0.554 | 0.563 | 0.567 | 0.569 |
| $q$         | 0.384 | 0.265 | 0.187 | 0.112 | 0.113 | 0.114 | 0.114 | 0.114 |
| $K$         | 0.052 | 0.052 | 0.168 | 0.232 | 0.262 | 0.272 | 0.278 | 0.280 |

## 7 Conclusion

In this paper we offer an explanation for the emergence of pay-as-you-go social security systems. Our story ties this development to the population shift from rural to urban areas, a migration that has its roots in increased life expectancy conditional on reaching age 60, and on technological progress in the city that outpaced that on the farm. This story fits the experience of the United States very well. We show that there is an initial steady state consistent with United States in the 1800s, with most people living on the farm and no social security system. Changes in life expectancy and technological progress in the city that are in line with those observed in the data initiate a transition to a new steady state. Along this transition path, a generation votes a social security system into place, which is supported throughout the transition and in the new steady state.

It is worth noting that the demographic changes alone would not lead to the rural-urban transition that the U.S. experienced. When we only change survival probabilities, social security does not emerge. Indeed, everybody remains on the farm. The key effect of this change is an increase in the capital stock because people save more anticipating a longer life.

When we only TFP changes social security does emerge but the rural/urban migration is not nearly as pronounced. Roughly 33% continue to live on the farm (in the data it is 23% and in our economy with changes in both survival probabilities and the TFP we get 16%). Furthermore, the social security tax is considerably higher than in the economy with both factors at work. This underscores the conclusion that the interaction between technology and demographics is a powerful impetus for social change.

## 8 Appendix

**Proof of Proposition 1:** The problem of a middle-aged agent is

$$\max_{a'} \left[ \log(I_m + a - \tau - a') + \beta\pi \log(I_o + (1+r)a' + \frac{2\tau}{\pi}) \right].$$

The first order condition for  $a'$  is given by

$$\frac{-1}{I_m + a - \tau - a'} + \frac{\beta\pi(1+r)}{I_o + (1+r)a' + \frac{2\tau}{\pi}} \leq 0.$$

Solving for an interior  $a'$  yields

$$a' = \frac{\beta\pi(1+r)[I_m + a - \tau] - (I_o + 2\tau/\pi)}{(1+r)(1+\beta\pi)},$$

which is positive if

$$\beta\pi(1+r)[I_m + a - \tau] - (I_o + 2\tau/\pi) \geq 0. \quad (33)$$

Since the right hand side of this inequality is decreasing in  $\tau$ , there exists a threshold tax level below which saving decision is positive and above which saving decision is zero. Finally, if  $a' > 0$ , then

$$\begin{aligned} \frac{\partial a'}{\partial \tau} &= \frac{-(\beta\pi(1+r) + \frac{2}{\pi})}{(1+r)(1+\beta\pi)} < 0, \\ \frac{\partial a'}{\partial a} &= \frac{\beta\pi}{1+\beta\pi} > 0, \end{aligned}$$

and

$$\frac{\partial a'}{\partial \pi} = \frac{\beta(1+r)^2[I_m + a - \tau] + (1+r)(1+\beta\pi)\frac{2\tau}{\pi^2} + \beta(1+r)(I_o + \frac{2\tau}{\pi})}{(1+r)^2(1+\beta\pi)^2} > 0.$$

**Proof of Proposition 2:** In order to compute the threshold tax level at which the savings decision becomes strictly positive, we solve for  $\tau$  in Equation (33). This yields

$$\hat{\tau} = \max \left\{ 0, \frac{\beta\pi(1+r)[I_m + a] - I_o}{2/\pi + \beta\pi(1+r)} \right\}.$$

If the threshold tax level is strictly positive, i.e. if  $\hat{\tau} > 0$ , then

$$\frac{\partial \tau}{\partial a} = \frac{\beta\pi(1+r)}{\beta\pi(1+r) + \frac{2}{\pi}} > 0.$$

**Proof of Proposition 3:** Consider now the problem of the middle-aged median voter. His optimal tax problem is given by

$$\max_{\tau} \left[ \log(I_m + a - \tau - a'(\tau)) + \beta\pi \log(I_o + (1+r)a'(\tau) + \frac{2\tau}{\pi}) \right].$$

The first order condition for  $\tau$  is

$$\frac{-(1 + \frac{\partial a'}{\partial \tau})}{I_m + a - \tau - a'(\tau)} + \frac{\beta\pi[(1+r)\frac{\partial a'}{\partial \tau} + \frac{2}{\pi}]}{I_o + (1+r)a'(\tau) + \frac{2\tau}{\pi}} = 0. \quad (34)$$

Remember that for any agent the threshold tax level was given by  $\hat{\tau} = \frac{\beta\pi(1+r)[I_m+a]-I_o}{2/\pi+\beta\pi(1+r)}$ . Hence, we would like to find parameter restrictions under which the optimal tax rate implied by Equation (34) is greater than, equal to and less than  $\hat{\tau}$ .

**Case 1:** Suppose the optimal tax rate of the median voter is greater than  $\hat{\tau}$  and as a result his savings decision is zero, i.e.,  $a' = 0$ . Then Equation (34) implies that the optimal tax rate is given by

$$\tau^* = \frac{2\beta[I_m + a] - I_o}{2\beta + \frac{2}{\pi}} \equiv \tilde{\tau}.$$

Note that if  $\frac{2}{\pi} > 1+r$ , then indeed  $\tilde{\tau} > \hat{\tau}$ . If  $\frac{2}{\pi} \leq 1+r$ , however, then  $\tau^* = \hat{\tau}$ .

**Case 2:** Consider now the other case, i.e. let  $a' = \frac{\beta\pi(1+r)[I_m+a-\tau]-(I_o+\frac{2\tau}{\pi})}{(1+r)(1+\beta\pi)}$  and  $\frac{\partial a'}{\partial \tau} = \frac{-(\beta\pi(1+r)+\frac{2}{\pi})}{(1+r)(1+\beta\pi)}$ . Then, the right hand side of Equation (34) becomes

$$\frac{(1 + \beta\pi)[\frac{2}{\pi} - (1+r)]}{[I_m + a - \tau](1+r) + [I_o + \frac{2\tau}{\pi}]}.$$

If  $\frac{2}{\pi} > 1+r$ , this expression is always positive and  $\tau^* = \hat{\tau}$ . If  $\frac{2}{\pi} < 1+r$ , it is always negative and  $\tau^* = 0$ . Finally, if  $\frac{2}{\pi} = 1+r$ , this expression is always zero and  $\tau^* \in [0, \hat{\tau}]$ . The saving choice is given by  $a' = \max\{0, \frac{\beta\pi(1+r)[I_m+a-\tau]-(I_o+\frac{2\tau}{\pi})}{(1+r)(1+\beta\pi)}\}$ . We summarize these case in Table A1.

TABLE A1

|                         | Case 1                  | Case 2                                                                      |
|-------------------------|-------------------------|-----------------------------------------------------------------------------|
| $\frac{2}{\pi} < 1 + r$ | $\tau^* = \hat{\tau}$   | $\tau^* = 0$                                                                |
|                         | $a' = 0$                | $a' = \max\{0, \frac{\beta\pi(1+r)[I_m+a]-I_o}{(1+r)(1+\beta\pi)}\}$        |
| $\frac{2}{\pi} > 1 + r$ | $\tau^* = \tilde{\tau}$ | $\tau^* = \hat{\tau}$                                                       |
|                         | $a' = 0$                | $a' = 0$                                                                    |
| $\frac{2}{\pi} = 1 + r$ | $\tau^* = \hat{\tau}$   | $\tau^* \in [0, \hat{\tau}]$                                                |
|                         | $a' = 0$                | $a' \in [0, \max\{0, \frac{\beta\pi(1+r)[I_m+a]-I_o}{(1+r)(1+\beta\pi)}\}]$ |

We next determine whether the median voter has higher utility in the first or the second case. Suppose  $\frac{2}{\pi} < 1 + r$ . The utility of a middle-aged agent in each is given by

$$U_{case1} = \log(I_m + a - \hat{\tau}) + \beta\pi \log(I_o + \frac{2}{\pi}\hat{\tau}),$$

and

$$U_{case2} = \log(I_m + a - a') + \beta\pi \log(I_o + (1+r)a').$$

Note that in the first case  $\hat{\tau}$  is a corner solution. In the second case, the asset choice is an interior solution over the interval,  $[0, \hat{a}]$ , where  $\hat{a} = \frac{\beta\pi(1+r)(I_m+a)-I_o}{(1+r)+\beta\pi(1+r)}$ . Since  $\hat{\tau} = \frac{\beta\pi(1+r)(I_m+a)-I_o}{\frac{2}{\pi}+\beta\pi(1+r)} > \hat{a}$ , consumption in the middle age is lower under  $\hat{\tau}$ . It also easy to show that  $(1+r)\hat{a} > \frac{2}{\pi}\hat{\tau}$ . Hence, consumption while old is also lower under  $\hat{\tau}$ . Therefore,  $U_{case2} \geq U_{case1}$ .

Suppose now  $\frac{2}{\pi} > 1 + r$ . Again the utility of a middle-aged agent in each case is

$$U_{case1} = \log(I_m + a - \tilde{\tau}) + \beta\pi \log(I_o + \frac{2}{\pi}\tilde{\tau}),$$

and

$$U_{case2} = \log(I_m + a - \hat{\tau}) + \beta\pi \log(I_o + \frac{2}{\pi}\hat{\tau}).$$

In the second case, the tax level is  $\hat{\tau}$ . This choice of tax level is also possible in the first case. Therefore, it must be that  $U_{case1} \geq U_{case2}$ .

Finally, suppose  $\frac{2}{\pi} = 1 + r$ . Then, the first case is a subset of the second one. We summarize Proposition 1 in Table A2.

TABLE A2

| Returns                 | Decisions                                                                                                                                                        |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\frac{2}{\pi} < 1 + r$ | $\tau^* = 0$<br>$a' = \max\{0, \frac{\beta\pi(1+r)[I_m+a]-I_o}{(1+r)(1+\beta\pi)}\}$                                                                             |
| $\frac{2}{\pi} > 1 + r$ | $\tau^* = \max\{0, \frac{2\beta[I_m+a]-I_o}{2\beta+\frac{2}{\pi}}\}$<br>$a' = 0$                                                                                 |
| $\frac{2}{\pi} = 1 + r$ | $\tau^* \in [0, \max\{0, \frac{\beta\pi(1+r)[I_m+a]-I_o}{2/\pi+\beta\pi(1+r)}\}]$<br>$a' \in [0, \max\{0, \frac{\beta\pi(1+r)[I_m+a]-I_o}{(1+r)(1+\beta\pi)}\}]$ |

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Figure 1 --- Population in Rural and Urban Areas

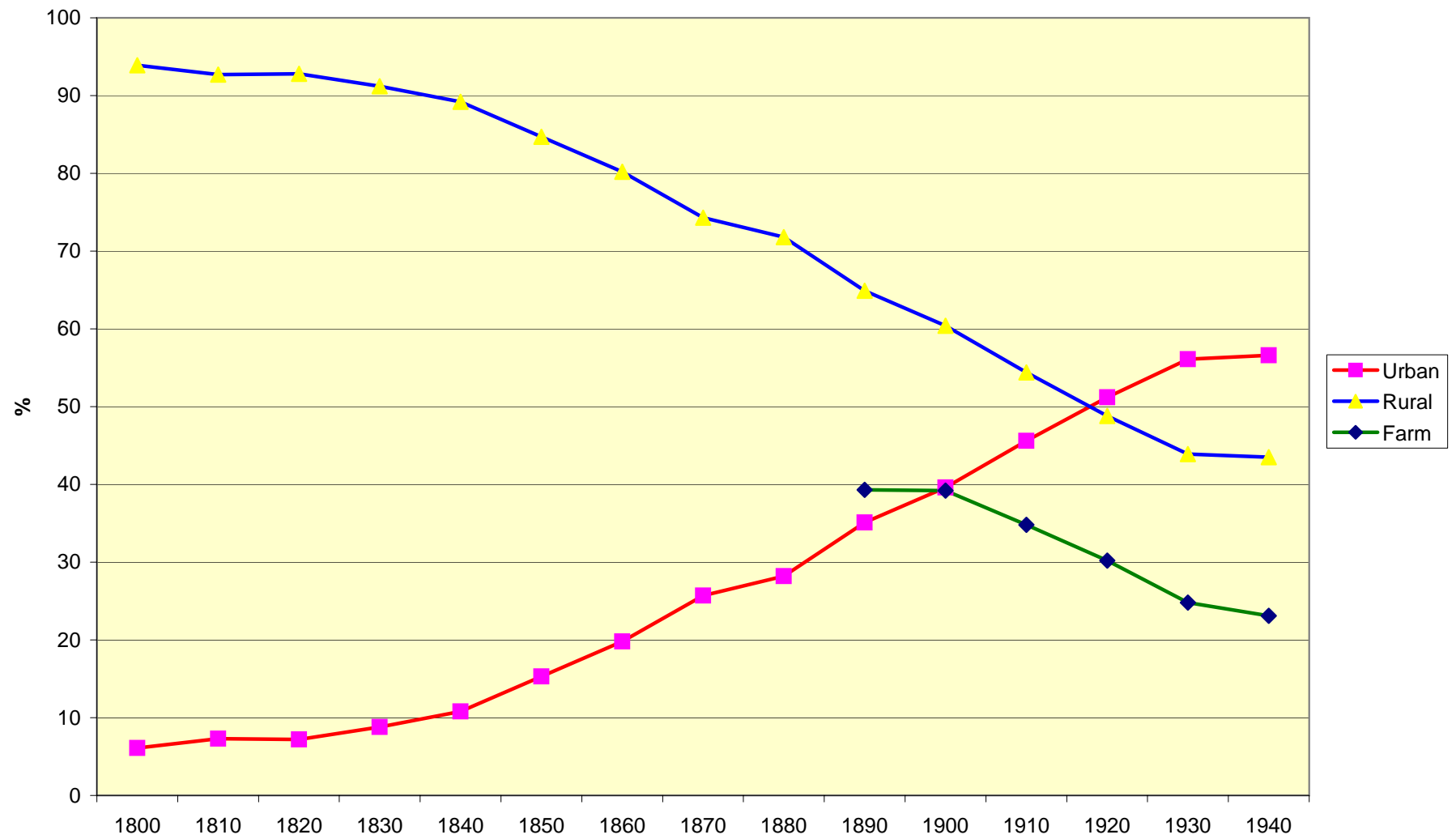
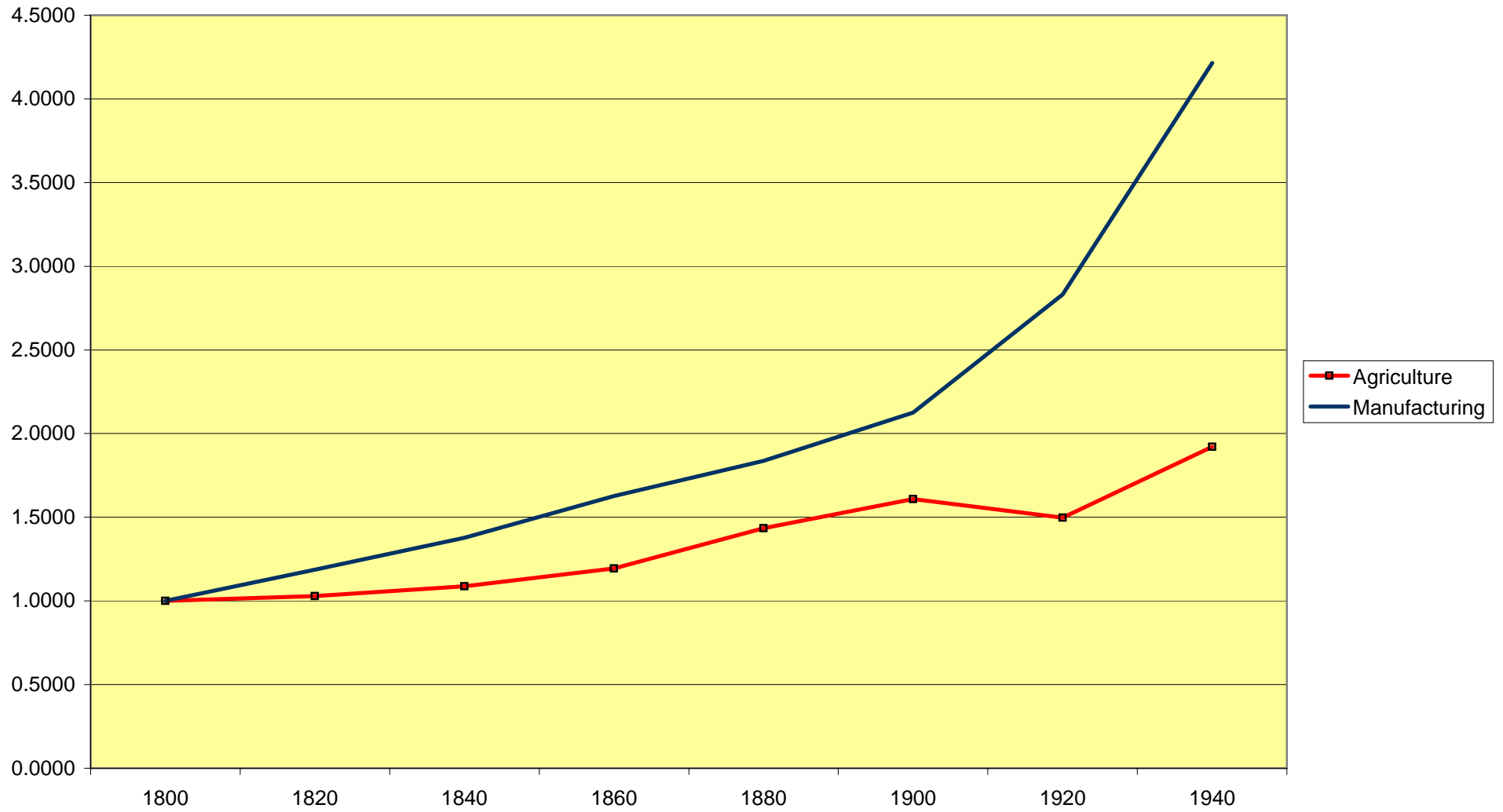


Figure 2 --- TFP in Agriculture and Non Agriculture



**Figure 3 --- Conditional Survival Probabilities**

